

- Let the matrices $A = \begin{pmatrix} 2 & 2 \\ 0 & 6 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 & 0 \\ 4 & 2 & 7 \\ 7 & 0 & 1 \end{pmatrix}$.
 - Find $A \times B$ if it exists.
 - Find $B \times A$ if it exists.
- Let $A = \begin{pmatrix} 2 & 5 & 7 \\ 9 & 5 & 7 \\ 2 & 8 & 9 \end{pmatrix}$ and $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.
 - Does the linear system $Ax = b$ have a unique solution, x ? How did you determine this?
 - What will be the dimension (the number of rows and columns) of b ?
- Find the first and second derivatives, $f'(x)$ and $f''(x)$, of the following functions.
 - $f(x) = 3x^3$
 - $f(x) = 5\sqrt{2x}$
 - $f(x) = 26x^4 + \ln(x) + 3$
 - $f(x) = \frac{2e^{0.5x}}{5}$
- Find the following partial derivatives for $f(x, y) = 15x^3 + 2xy^2 + 8y^2 + 8$.
 - $\frac{\partial f}{\partial x} = f_x$
 - $\frac{\partial f}{\partial y} = f_y$
 - $\frac{\partial^2 f}{(\partial x)^2} = f_{xx}$
 - $\frac{\partial^2 f}{(\partial y)^2} = f_{yy}$
 - $\frac{\partial^2 f}{\partial x \partial y} = f_{xy}$
- Suppose a competitive firm sells 2 products at market prices p_1 and p_2 and faces a cost function of $C(q) = 8q_1^2 + 3q_2^2 + 10$. The managers must produce an optimal quantity of each product to sell, q_1 and q_2 .
 - Write an expression for the profits of the firm.
 - Use the first order conditions to find potential expressions for the profit maximizing output, q_1^* and q_2^* .
 - Confirm that any values from b. are indeed profit maximizing.
 - If $p_1 = 1,600$ and $p_2 = 300$, what is the optimal output (q_1^* and q_2^*) and profit for the firm?

1. Let the matrices $A = \begin{pmatrix} 2 & 2 \\ 0 & 6 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 & 0 \\ 4 & 2 & 7 \\ 7 & 0 & 1 \end{pmatrix}$.

- Find $A \times B$ if it exists.
- Find $B \times A$ if it exists.

$$AB = \begin{bmatrix} 2 & 2 \\ 0 & 6 \\ 1 & 2 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 2 & 3 & 0 \\ 4 & 2 & 7 \\ 7 & 0 & 1 \end{bmatrix}_{3 \times 3}$$

does not exist.

$$BA = \begin{bmatrix} 2 & 3 & 0 \\ 4 & 2 & 7 \\ 7 & 0 & 1 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 2 & 2 \\ 0 & 6 \\ 1 & 2 \end{bmatrix}_{3 \times 2}$$

$$= \begin{bmatrix} 4 & 22 \\ 15 & 34 \\ 15 & 16 \end{bmatrix}$$

2. Let $A = \begin{pmatrix} 2 & 5 & 7 \\ 9 & 5 & 7 \\ 2 & 8 & 9 \end{pmatrix}$ and $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

- Does the linear system $Ax = b$ have a unique solution, x ? How did you determine this?
- What will be the dimension (the number of rows and columns) of b ?

$$a) |A| = \begin{vmatrix} 2 & 5 & 7 \\ 9 & 5 & 7 \\ 2 & 8 & 9 \end{vmatrix} = 77 \neq 0$$

Yes $Ax = b$ has a unique solution

$$b) 3 \times 1$$

3. Find the first and second derivatives, $f'(x)$ and $f''(x)$, of the following functions.

- $f(x) = 3x^3$
- $f(x) = 5\sqrt{2x}$
- $f(x) = 26x^4 + \ln(x) + 3$
- $f(x) = \frac{2e^{0.5x}}{5}$

$$a) f'(x) = 9x^2$$

$$f''(x) = 18x$$

$$b) f'(x) = \frac{5 \cdot 2}{2\sqrt{2x}} = \frac{5}{\sqrt{2x}}$$

$$f''(x) = 5 \left(-\frac{1}{2}\right) (2x)^{-3/2} (2) = -\frac{5}{(2x)^{3/2}}$$

$$c) \quad f'(x) = 104x^3 + \frac{1}{x}$$

$$f''(x) = 312x^2 - \frac{1}{x^2}$$

$$d) \quad f'(x) = \frac{2}{5} e^{0.5x} (0.5) = \frac{1}{5} e^{0.5x}$$

$$f''(x) = \frac{1}{10} e^{0.5x}$$

4. Find the following partial derivatives for $f(x, y) = 15x^3 + 2xy^2 + 8y^2 + 8$.

a. $\frac{\partial f}{\partial x} = f_x$

b. $\frac{\partial f}{\partial y} = f_y$

c. $\frac{\partial^2 f}{(\partial x)^2} = f_{xx}$

d. $\frac{\partial^2 f}{(\partial y)^2} = f_{yy}$

e. $\frac{\partial^2 f}{\partial x \partial y} = f_{xy}$

$$a) \quad f_x = 45x^2 + 4xy$$

$$b) \quad f_y = 4xy + 16y$$

$$c) \quad f_{xx} = 90x + 4y$$

$$d) \quad f_{yy} = 4x + 16$$

$$e) \quad f_{xy} = 4y$$

5. Suppose a competitive firm sells 2 products at market prices p_1 and p_2 and faces a cost function of $C(q) = 8q_1^2 + 3q_2^2 + 10$. The managers must produce an optimal quantity of each product to sell, q_1 and q_2 .
- Write an expression for the profits of the firm.
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$$a) \quad R = p_1 q_1 + p_2 q_2$$

$$\text{Profit} = p_1 q_1 + p_2 q_2 - 8q_1^2 - 3q_2^2 - 10$$

$$b) \quad \frac{\partial P}{\partial q_1} = p_1 - 16q_1 = 0$$

$$q_1 = \frac{p_1}{16}$$

$$\frac{\partial P}{\partial q_2} = p_2 - 6q_2 = 0$$

$$q_2 = \frac{p_2}{6}$$

$$c) \quad \frac{\partial^2 P}{\partial q_1^2} = -16 < 0$$

$$\frac{\partial^2 P}{\partial q_2^2} = -6 < 0$$

maximum.

$$d) \quad q_1 = \frac{1600}{16} = 100$$

$$q_2 = \frac{300}{6} = 50$$